Wages, productivity, employment, inflation and distribution are linked by accounting identities.

Labor productivity (or just productivity, for short) is defined as the amount produced, divided by the amount of labor used to produce it. The labor share is defined as the fraction of total income paid out to labor. We can use these two *identities* to analyze changes in output, employment, wages and prices. This doesn't tell us what will happen in the economy, but it does tell us something about what *can* happen. Using these identities also helps us describe developments in the economy more precisely, and clarifies what assumptions are needed for various stories or predictions about the economy to be true.

There is a mathematical rule that lets us convert equations to linear form, which is easier to work with.

A **linear equation** is one in which the variables are only added or subtracted. None of the variables are multiplied by each other, and none are raised to a power (that is, there are no expressions like x^2). Linear equations are generally easier to work with, so it's convenient to be able to change other kinds of equations to linear ones if possible.

One useful tool for making linear equations is: If $a = b * c$ then

$$\textit{percentage change in a} \approx \textit{percentage change in b} + \textit{percentage change in c} \tag{1}$$

Similarly, if $a = b/c$ then

$$\textit{percentage change in a} \approx \textit{percentage change in b} - \textit{percentage change in c} \tag{2}$$

The Greek letter Δ (delta) is often used to mean the change in a variable. So to save space, I will write %Δ when I mean "percent change in..." For example " %Δ employment" means "percent change in employment."

So we can rewrite Equation 1 as:

$$\%\Delta a \approx \%\Delta b + \%\Delta c$$

This is linear – the variables are simply added. Whereas $a = b * c$ is not linear, since the variables are multiplied. Note that the original equation described the *levels* of the variables, while the new, linear one describes the *changes* in them.

This works the same if we have more than two variables on the right hand side.

Labor productivity is defined as output divided by employment.

When economists talk about "productivity", they mean either **labor productivity** or **total factor productivity.** Labor productivity is the output produced by a given amount of labor; total factor productivity is the amount of output produced by a given amount of labor and capital. Total factor productivity is important for economic theory, but it is hard to apply in practice, since measuring capital is difficult and you need to make additional assumptions about how the labor and capital are combined. For most practical purposes, labor productivity is more relevant. Whenever I refer to "productivity" here, I mean labor productivity.

Labor productivity is defined as output divided by the amount of labor used. Labor can be measured either in hours of work, or number of people employed. Here we will measure it by number of people employed. So labor productivity is defined by:

$$productivity = \frac{output}{employment} \tag{3}$$

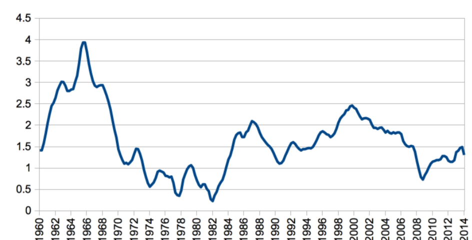

Figure 1: US labor productivity growth rates, 5-year moving averages. In recent years, productivity has grown at less than 1.5 percent per year, which is lower than in much of the postwar period.

We can measure productivity for the economy as a whole, for an industry or sector, or for a single business. If we are measuring it for the economy as a whole, then "output" is GDP; for an industry or business, it is value added. When we are talking about changes in productivity, we normally measure output in real (or inflation-adjusted) terms.

We can rearrange Equation 3 to get

$$output = employment * productivity$$

In other words, total production in an economy (or an industry or business) is equal to the number of people employed, times the average amount produced by each one.

The change in employment over some period of time is equal to the change in output minus *the change in labor productivity.*

We can analyze changes in employment in terms of changes in output and productivity. Using Equation 2, we can write:

$$\%\Delta employment \approx \%\Delta output - \%\Delta productivity \tag{4}$$

The percent change in employment is equal to the percent change in output minus the percent change in employment. For example, in 2014, total employment in the US rose by 2.2 percent, output rose by 2.9 percent, and productivity rose by 0.7 percent. (This is an exceptionally low rate by historical standards.) We can apply this equation to a change over one year or over several years. But if we apply it to a very long period (say, 50 years), the approximation may be less accurate.

Equation 4 is an accounting identity: it is true by definition. But it still shows us a couple of things that might not be obvious.

First of all, changes in employment can be due to either changes in total output, or to changes in productivity – that is, either changes in how much is produced, or in how much labor is used for a given amount of production. Over short periods, changes in output growth are much more important. For example, Table 1 shows the average annual change in employment during the expansion of 2002-2007 and the recession of 2008-2009.

Period	Employment	Output	Productivity
2002-2007	0.8%	2.7%	1.9 %
2008-2009	-3.1%	-1.5%	1.6%

Table 1: Average Annual Change in Employment, Output and Productivity

Employment grew at an average rate of 0.8 percent per year over 2002-2007, and fell at a rate of 3.1 percent per year during 2008-2009. This difference is entirely explained by the fact that output was rising during the first period, and falling in the second period. As you can see, labor productivity actually grew somewhat slower during the recession than during the expansion, but the change is quite small compared with the changes in output and employment growth.

Over long periods, faster labor productivity growth could contribute to slower employment growth. This is called "technological unemployment," but it is not clear that it is a real problem.

Over longer periods, however, changes in the speed of labor productivity growth may be more important. The second thing that Equation 4 tells us is that if output is growing at a constant rate, than faster productivity growth must men slower growth in employment. If productivity grew fast enough, you might even see a situation where output continued to grow while employment fell.

The idea that rapid improvements in labor productivity might lead to a fall in employment is a familiar one in the media and in policy discussions. It is often referred to as **technological unemployment** – the idea that "robots will take our jobs." Obviously, there are many specific cases of work jobs that become obsolete through technological change. But Equation 4 helps us think about this possibility more systematically. To begin with, it highlights the point that a prediction that technological progress will reduce employment is simply a claim that we will see an acceleration of productivity growth but not of GDP growth. But this raises two questions. First, productivity growth has been slowing down in recent years, not accelerating. Since 2000, productivity growth has averaged barely one percent a year, compared with around 1.5 percent per year during the period between 1950 and 2000 (and as high as 3 percent a year during the 1960s). So the "robots will take our jobs" story is not just extrapolating from what is already happening; someone telling this story has to explain why the recent trend of declining productivity will reverse itself. Second, Equation 4 makes it clear that faster productivity growth can lead to lower employment *or* to faster growth in output. Someone telling the "robots will take our jobs" story also has to explain why faster productivity growth will not simply lead to faster growth of GDP. After all, 120 years ago most Americans worked in agriculture. Technological change has resulted in the disappearance of almost all of those jobs. But the result has not been mass unemployment, but rather increased production in the other (secondary and tertiary) sectors of the economy.

While the statistics are not decisive either way, there is some evidence that labor productivity rises faster when output is growing more rapidly. In this case, we might see the opposite of technological unemployment – employment and productivity moving together. For example, the early 1930s, when employment fell very steeply, labor productivity actually declined – one of the only periods on record when this occurred. And when employment rose in the recovery from the Depression, productivity rose as well. Note that in this case Equation 4 was still true – as an accounting identity, it is always true – but the big changes in output overwhelmed the effect of productivity on employment.

The technological unemployment issue is an example of why accounting identities are useful. They can't prove that a certain story about the economy is true. But they can clarify what that story means, and show what assumptions it involves.

The labor share is the fraction of total income going to wages.

Another useful accounting identity is that the **labor share** is the fraction of total income that comes as wages and salaries. In the simplest story, all income is either labor income or capital income. But we can talk about the labor share even if there are other kinds of income. It simply means the fraction of total income that is received by labor. We can write:

$$\textit{labor share} = \frac{\textit{total wages}}{\textit{output}} \tag{5}$$

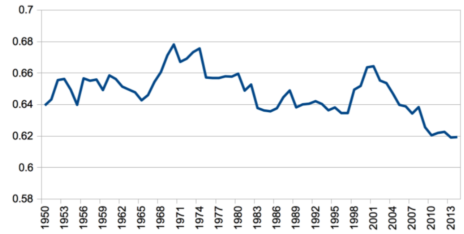

Figure 2: US labor share. During the 1980s, and again over the past 15 years, there was a fall in the share of national income going to labor.

As with productivity, we can apply this to the economy as a whole or to a particular sector or industry.[1] Wages here include fringe benefits, like health insurance or pension contributions. In this equations, wages and output are measured in *nominal* terms.

[1] Note that in the national accounts, the labor share in government and nonprofits is 100% by definition. So if we look just at the business sector, the labor share will always be somewhat lower than for the economy as a whole.

The change in the average nominal wage is equal to the change in average productivity, plus the inflation rate, plus the change in the wage share.

Equation 5 is an accounting identity. We also know that total wages equals the average wage times total employment, the real wage equals the nominal wage divided by the price level, and the change in the price level is inflation.

Substituting these identities into Equation 5 and applying the linear approximation gives us[2]

[2] I show exactly how this is derived in the note at the end.

$$\%\Delta \textit{nominal wage} \approx \%\Delta \textit{productivity} + \%\Delta \textit{prices} + \%\Delta \textit{wage share}$$

In other words, the percentage increase in the average nominal wage must be equal to the sum of the percentage increases of labor productivity, the price level, and the wage share. And since inflation is jsut the percentage change in the price level, we can rewrite this as:

$$\%\Delta \textit{nominal wage} \approx \%\Delta \textit{productivity} + \textit{inflation} + \%\Delta \textit{wage share} \tag{6}$$

To think about what this means, imagine a business that for whatever reason decides to increase wages. What can happen as a result? It might be that profits will fall – that is a rise in the labor share. It might be that it will increase its prices – over the economy as a whole, that is the same as inflation. Or it might be that the higher wages will cause the business to become more productive, perhaps because workers will be more concerned about losing such a good job or because they will feel a greater sense of loyalty. Any of these outcomes are possible. But what we know for sure is that a one percent increase in wages *must* result in some combination of a higher wage share, higher prices, and/or higher productivity, that add up to one percent. This is true both at the level of an individual business and for the economy as a whole.

Equation 6 lets us think systematically about many things that happen in the economy. For instance, what happens if labor productivity grows more rapidly, while nominal wage growth is unchanged? The equation says that in this case either inflation or the wage share must fall. The fact that more rapid productivity growth is *deflationary* – tends to lead to lower prices – is not obvious, but the equation makes it clear.

Macroeconomic theory often assumes that the wage share is fixed. This implies that the increase in real wages (the increase in nominal wages less inflation) must be just equal to the growth of productivity. But in the real world, this is often not the case. Over the past 15 years, nominal wages have increased by an average of 2.9 percent a year, inflation has averaged 2.4 percent a year, and productivity has increased by 1.1 percent per year. This means that the labor share has fallen by a bit over half a percent per year. ($2.9 - 2.4 - 1.1 = -0.6$.) While this might not seem like much, over 15 years that adds up to a 9 percent total decrease – a substantial fall in the share of income going to workers.[3]

[3] This is a *9 percent decrease*, not a *9 percentage point* decrease.

We can turn Equation 6 around and ask what happens when productivity increases.

$$\%\Delta \mathit{productivity} \approx \%\Delta \mathit{nominal\ wage} - \%\Delta \mathit{prices} - \%\Delta \mathit{wage\ share}$$

Again, think of an individual business: Rising labor productivity means they are now able to produce the same quantity of goods with fewer workers. Let's say the number of workers required per unit of output has fallen by 10 percent. What happens? Either the remaining, more productive workers can each be paid 10 percent more; or the company can cut its prices by 10 percent; or the gains from increased productivity can go to higher profits (i.e. a lower labor share). Again, the accounting identity doesn't tell us which of these outcomes will happen. But it does tell us that one of them – or some combination – *must* happen whenever productivity rises.

Note: Deriving Equation 6

We know that the wage share is equal to total nominal wages divided by nominal output. This implies that

$$\textit{total nominal wages} = \textit{nominal output} * \textit{wage share}$$

We know that nominal output is equal to real output times the price level. So substitute that in:

$$\textit{total nominal wages} = \textit{real output} * \textit{price level} * \textit{wage share}$$

We know that real output is equal to employment times productivity, so substitute that in:

$$\textit{total nominal wages} = \textit{productivity} * \textit{employment} * \textit{price level} * \textit{wage share}$$

Finally, we know that the average wage is equal to total wages divided by total employment, so we divide both sides by employment to get:

$$\textit{nominal wage} = \textit{productivity} * \textit{price level} * \textit{wage share}$$

And now we apply the linear approximation of Equation 1, and that gets us Equation 6.